# Electron and X-Ray Diffraction Report Requirements

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## Abstract-

The diffraction of electrons through a thin graphite target was performed, this diffraction directly exposes the spacing between planes in a braggs scattering experiment. By measuring the projected rings and their spacing the spacing between planes in the graphite lattice structure can be determined. Herein, the acquired values were 0.0496 and 0.0347 respectively. Their ratio was 1.43, or approximately the square root of two, as compared to an expectation value of 1.73. This discrepancy leaves a error value of 17.44% which is quite substantial and can be attributed to equipment quality, environmental disturbances and the possible breakdown of some fundamental assumptions made in theory. These results indicate a square lattice structure, but the lattice structure provided in the experiment is designed to be hexagonal, this deviation is discussed in the conclusion.

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**Introduction-**

This experiment demonstrates the wave particle duality of electrons and other quantum particles by firing a semi coherent electron beam at a graphite diffraction target, thereby splitting the beam in predictable ways proportional to the Bragg's spacing of the graphite crystal lattice.

Early in the history of quantum physics and particle science, the dual nature of photons posed a substantial challenge to contemporary thinkers. Compton showed the particle nature of light in the 1950s with his experiments in X-ray diffraction through crystal lattices. By demonstrating instantaneous momentum transfer between light particles and the atoms of the crystal lattice, Compton was able to explain the scattering effects he observed when varying the angle of attack of his X-ray beam. The work by compton helped to verify the particle side of the particle wave duality commonly discussed in quantum physics today. The particle component of the theory breaks down however, when diffraction, polarization and other wave behavior of light comes into play. One example of the failure of the corpuscular explanation lies in Einstein's description of the photoelectric effect, and even in earlier examples, such as Hertz and his polished spark gap transmitter systems. Einstein described the energy of the "wave packets" he called, photons, in terms of wavelength and frequency, thus making them dependent on a wave characteristic description of their underlying structure. This description by Einstein is part of what sparked the major debate in the $20^{th}$ century on the true nature of light and matter. Are particles waves and waves particles? Is light both particle and wave or something entirely new? All questions that would drive scientific progress in the $20^{th}$ century. One approach to this mysterious dual nature of light, would be that of Louis de Broglie and his doctoral dissertation on the wave particle duality of light. De Broglie used the description previously espoused by max Planck in his characterization of black body radiation. That is, de Broglie was able to relate Planck's quantized model of light to the wave nature of light espoused by Einsteins special relativity.

Mathematically, de Broglie merely assumed that Einstein's description would be valid for particulate light as well as wave based light in a derivation below,

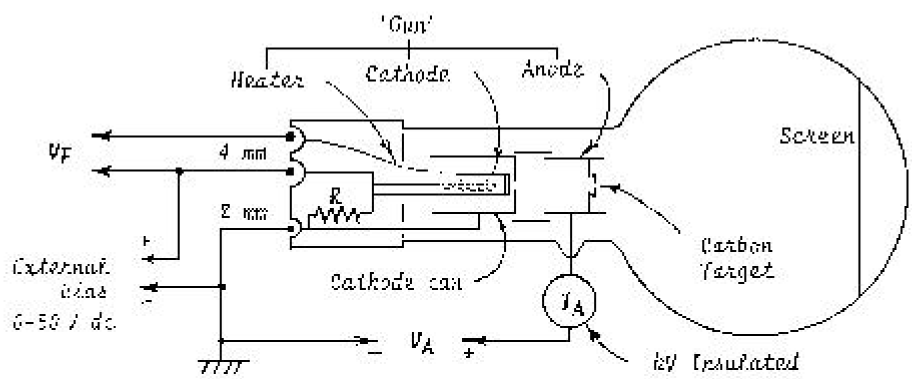
$$E = hf$$

$$p = hv/c = h/\lambda.$$

Thus, was born the concept of a "de Broglie" wavelength. That is, all objects, all matter has some wavelength associated with it, that depends only on planck's constant, the object's momentum, and the speed of light. This concept proved especially potent for bridging the gaps in theoretical understanding in de Broglie’s time. Generally speaking, for objects moving at classical velocities and with masses on the classical scale, the de Broglie wavelength remains so small as be virtually immeasurable. But for very small ,very fast things, like electrons or x-rays, the de-Broglie wavelength becomes increasingly apparent and fundamentally alters the behavior of these “particles” to what can be described as a “matter wave” as in the modern quantum mechanical interpretation. For electrons specifically, the de Broglie wavelength spans several angstroms, allowing them to display many properties of both waves and particles, as their diameter is much smaller than their de Broglie wavelength and varies in its proportionality with the velocity of any given electron. Similarly, and as a more extreme example, X-rays have a de Broglie wavelength comparable to an electron traveling at near the speed of light.

Eventually, the understanding gained by use of de Broglies relatively simple assumption would prompt the famous double slit experiment, wherein two slits with widths comparable to the wavelength of the “wavicles” to be diffracted or deflected are placed between an electron or light source and a fluorescent display. The double slit experiment allows the wave and particle nature of a particle object, electrons, x-rays, and even photons of other wavelengths, to be teased out depending on the specific conditions of the experiment. Under appropriate conditions of electron production, wave behavior can be seen, and simultaneously under other appropriate experimental conditions, particle nature can be seen; thus providing an experimental basis for the concept of wave particle duality as it has come to be know today in quantum mechanics and modern physics at large.

In this study, diffraction of electrons through thin graphite targets is used to demonstrate the wave nature of electrons, by displaying the diffraction patterns characteristic of waves difffracted by slits in a evacuated experimental containment, similar to the experiments performed by Davisson and Germer in 1927. Differences between their experiment and the one herein include their use of a nikel target, and differing availability and quality of precision voltage sources. A picture of the experiment is in figure 1. below.



**Theory-**

Diffraction of electrons through a target in a cathode ray tube involves the production of fast electrons by "boiling" them off a heated filament and then using accelerating potentials across plates to yield a stream of relatively homogeneous electrons with an approximately common velocity. These roughly homogeneous electrons pass through a thin diffraction target made of graphite resulting in diffraction effects as the target interferes with the wave like electrons.

To relate the velocity of the accelerated electrons to the accelerating voltage, the classical kinetic energy relationship can be stated as follows,

$$K= \frac{1}{2}mv^2=eV_a \quad .$$

Following de Broglies footsteps, we first state the momentum relation for photons,

$$p=mv=\frac{h}{\lambda} \quad .$$

Then we solve the classical kinetic energy relationship for the velocity term and substitute it into the photon momentum relation to solve for wavelength as below,

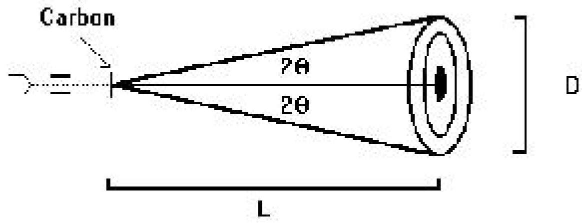
$$\lambda=\frac{h}{\sqrt{2V_A me}} \quad .$$

Where h is Planck's constant, with a known value of 4.136 x 10^-15 electron-volts*seconds, m is the mass of an electron or 9.109 x 10^-31 kilograms, and the electron's elementary charge, e, is 1.602 x 10^-19 Coulombs. Plugging in the constants yields the value for the de Broglie wavelength as:

$$\lambda=\sqrt{\frac{150}{V_A}}$$

Where the wavelength is in units of Angstroms. Thus, we have retraced De Broglie's steps in deriving the wavelength of everything.

Furthermore, the De Broglie wavelength can be verified by examining the diffraction of electrons as demonstrated by the spacing between planes seen in braggs scattering experiments. The spacing is dependent on the angle and path length of the accelerated electrons, thus a small angle approximation can be used to predict the positions and spacing of the observed planes, as shown in the figure below.




The associated angle approximation takes the form,

$$2\theta = \sin(2\theta) = \tan(2\theta)$$

where the length L is 0.042 millimeters.

Additionally, the rings that appear on the fluorescent screen will have diameters differing depending on their angle of separation.

When the angles are small, which they are in this instance, a small angle approximation can be used thusly,

$$\theta = \frac{D}{4L} \quad .$$

Using this new angle approximation, braggs law can be applied as,

$$n\lambda = 2d*\sin\theta$$

which solved for d where n = 1 and the using the previously calculated de Broglie wavelength yields,

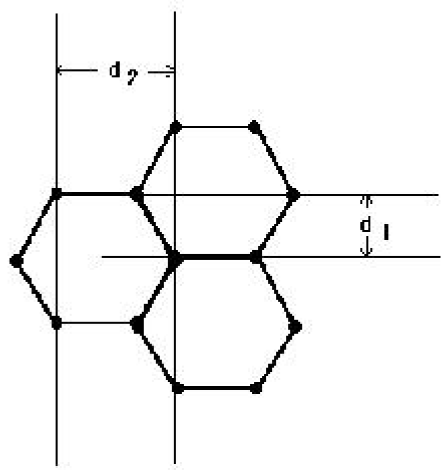
$$d = \frac{2l\sqrt{(150)}}{D\sqrt{(V_a)}} \quad .$$

Using this distance, and the above values, an approximate slope for the graph of diameter versus the inverse root of accelerating voltage, accurate for the small angle approximation, can be found as,

$$M = \frac{2L\sqrt{(150)}}{d} \quad .$$

This graph allows d to be found experimentally by averaging over all the data points on the graph. The value of ‘d’ will be in units of angstroms.

The reason Braggs scattering is seen in the experiment in due to diffraction and scattering effects on the electrons passing through the graphite target, an excellent example of both wave and particle characteristics playing a role in real world phenomena simultaneously. The exact diffraction patter observed and the plane spacing seen in Bragg scattering experiments depends on the spacing of the lattice structure. Graphite is a stacked set of atomically thin sheets of carbon with hexagonal lattice structure, as shown below.




The crystal lattice spacing, directly determines the intensity of scattering phenomena, and diffraction effects. In short, the crystal lattice spacing directly determines the ration of inner to outer diameters of the planes seen in the scattering experiment. The expected value of this ratio is $\sqrt{3}$ . Were it would be one in the case of a square lattice structure. The calculation of this ratio from first principles can be seen in the image below.

$$\theta = \frac{(n-2)\,180}{6}$$

$$\theta = 120^\circ$$

$$\phi = 90^\circ - (180^\circ - \theta)$$

$$\phi = 30^\circ$$

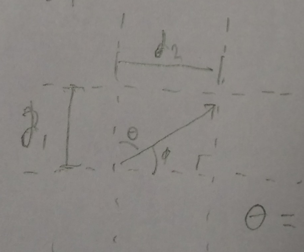
$$d_1 = s\cos\phi$$

$$d_2 = s + s\sin\phi = s(1+\sin\phi)$$

$$\frac{d_2}{d_1} = \frac{s(1+\sin(30^\circ))}{s\cos(30^\circ)} = \frac{1+\frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

$$\frac{d_2}{d_1} = \frac{\left(\frac{3}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{3(2)}{2\sqrt{3}}$$

$$\frac{d_2}{d_1} = \sqrt{3}$$



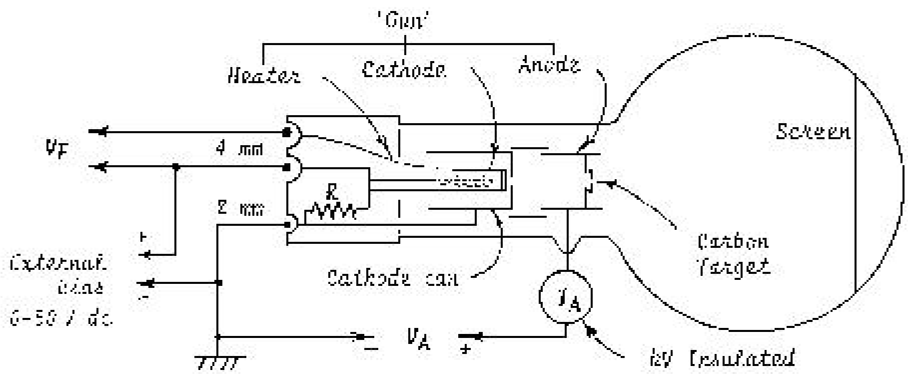

$\theta = \phi = 45^\circ$

For a cube

$$\boxed{\frac{d_2}{d_1} = 1}$$

**Experimental Procedure-**

The diffraction and scattering experiment was setup at below.

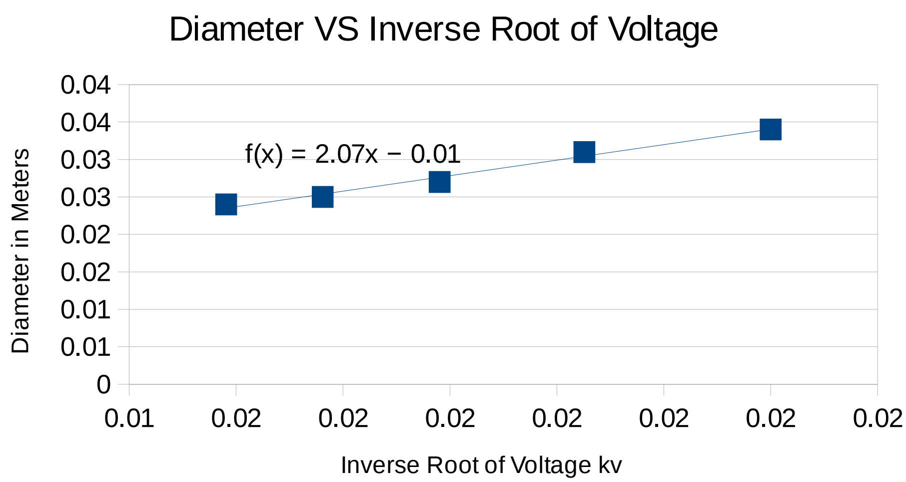


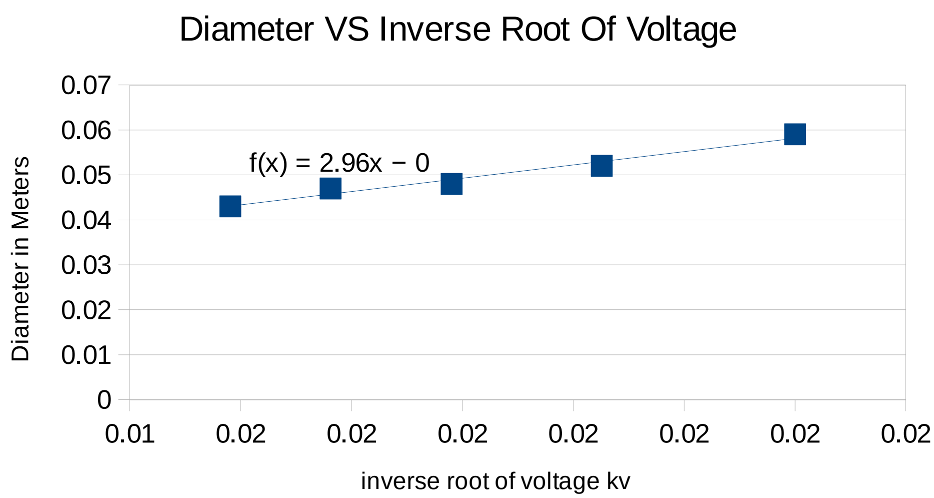

Wire the cathode ray tube correctly as above. A wired power supply and anode voltage source should be connected and set as above. One should be careful that the anode current does not exceed 0.2 milliAmperes, adjusting down if necessary, as the graphite target is very thin, and susceptible to penetration under over current conditions.  The anode voltage should be set at 5 incremements between 2500 and 5000 volts during the experiment. At each value of anode voltage there should be a an inner and outer ring coalescing as an image on the fluorescent screen at the end of the bulb. The rings can be quite dim, so all lights in the room ought to be dimmed or disabled before data is recorded. The diameter of both rings should be recorded for each anode voltage value. The length from the carbon target to the screen for the experiment is 0.042 centimeters.  After recording the values of ring diameter the Bragg Plane spacings can be solved for using available relationships. There will be a Bragg spacing, unique for each ring, inner and outer. Once these values are found, a ratio between them can be calculated and compared to a theoretical value for the carbon lattice, as in the theoretical calculations above.

## Graphs and Data-

| Voltage (V) | Diameter of Outside Ring (meters) |
|---|---|
| 0.02 | 0.059 |
| 0.01825742 | 0.052 |
| 0.01690309 | 0.048 |
| 0.01581139 | 0.047 |
| 0.01490712 | 0.043 |

| Voltage (V) | Diameter of Inside Ring (meters) |
|---|---|
| 0.02 | 0.034 |
| 0.01825742 | 0.031 |
| 0.01690309 | 0.027 |
| 0.01581139 | 0.025 |
| 0.01490712 | 0.024 |





Diameter VS Inverse Root Of Voltage
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
Diameter in Meters
f(x) = 2.96x − 0
0.01
0.02
0.02
0.02
0.02
0.02
0.02
0.02
inverse root of voltage kv

**Calculations-**

The momentum of a photon is,

$$p = \frac{h}{\lambda}$$

and the kinetic energy,

$$K = \frac{1}{2} m v^2 = e V_a$$

One can solve for velocity.

$$v = \sqrt{\left(\frac{2 e V_a}{m}\right)}$$

and thus, for wavelength,

λ = h/P

Where momentum can also equal,

P = mv

After substitutions wavelength can be found,

$$\lambda = \frac{h}{m * \sqrt{\left(\frac{2 e V_a}{m}\right)}}$$

$$\lambda = \frac{h}{\sqrt{(2 m e V_a)}}$$

Where h = 6.626x10^-34 Js, m = 9.109x10^-31 kg, and e = 1.602x10^-19 C.

Plugging in these constants and solving,

$$\lambda = \sqrt{\left(\frac{1.50 * 10^{-18}}{V_a}\right)}$$

Plugging this into Braggs law where n = 1,

$$n\lambda = 2d\sin\theta$$

along with the geometric calculation,

$$\tan(2\theta) = D/2L$$

It is true that,

$$\tan 2\theta = \sin 2\theta = 2\theta$$

and thus,

$$\theta = D/4L$$

Plugging this back into Bragg's law for n=1.

$$\lambda = 2d\theta$$

$$\lambda = 2d(D/4L)$$

$$d = (2\lambda L)/D$$

Substitute for the solution of λ yielding,

$$d = \frac{2L}{\sqrt{(150)}} / \left(D * \sqrt{(V_a)}\right)$$

For the slope of D vs. $(1/\sqrt{V_A})$ ,

$$M = \frac{2L\sqrt{(150)}}{d}$$

Solving for the Bragg spacing, d, yields,

$$d = \frac{2l\sqrt{(150)}}{D\sqrt{(V_a)}}$$

**Analysis and Conclusions-**

The values of d_1 and d_2 that were found by experiment were .0496 Angstroms and .0347 Angstroms respectively. Their ratio turned out to be 1.43, which is closer to the square root of 2, not the square root of 3. These values represent an error of 17.45 %, a substantial discrepancy. This discrepancy between expected theoretical values, and observed experimental values possibly arises from the effects of the earth's magnetic field, electronic systems and other sources of local electromagnetic disturbance that could cause alterations in the paths, or flight speeds of the accelerated electrons as they pass through the graphite target. Furthermore, the questionable age and quality of the experimental apparatus could be to blame for the relatively large error. Anything from misalignment of accelerating plates or the graphite target, to quality assurance failures of the target itself, or discrepancies in the actual path length and the theoretical value of path length used for calculations herein could be contributing to the observed error. It is also possible, that the angle of scattering is not as small as assumed, and more electrons are lost or diffracted at more extreme angles than can be reasonably accounted for using the small angle approximation above. This increase in diffraction angle would likely result in larger diffraction rings that expected, and would increase uncertainty of the experimental results by spreading the expected diameter over a larger region. In this case, where the assumptions of small angle diffraction and relatively tight diffraction ring sizes are not met, the graphs in the calculation section would likely suffer from inaccuracies and have a larger standard deviation to boot. Larger ring sizes correspond to greater uncertainties and greater deviation from the mean. As each ring represents a roughly Gaussian distribution of impacting diffracted electrons, the width of the rings is directly correlated to experimental uncertainty and the standard deviation in the data. Yet another probable source of error was the use of linear trend lines to fit nonlinear data. By using the slope of those trend lines to calculate the ratio of the lattice spacings, some error was introduced as a result of less than ideal curve fitting and a relatively small number of data points. The precision of the experiment could be improved by increasing the number of data points, and repairing the experimental apparatus to have appropriate plate alignment.

**Bibliography-**

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*Brian Greene, The Elegant Universe, page 104 "all matter has a wave-like character"*

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