Modern Physics: Electrostatic Deflection

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Abstract:

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Introduction:

The purpose of this research is to explore the deflection of electrons in a cathode ray tube by charged plates and an external EM field. The modified Thompson cathode ray tube apparatus was used to recreate Thompson's experiment for determining the charge to mass ratio of the electron. The equipment used consisted of a TEL 525 cathode ray tube and a 502 Helmholtz coil for electric and magnetic field production respectively. The deflection of the electron using known field values allowed the analysis of time of flight, the path geometry and the deflection of the cathode ray as a function of voltage affecting discovery of the charge to mass ratio as well as calculation of the particle velocity.

The path of charged particles can be influenced by electric and magnetic fields, as in the case of cathode ray tube televisions and oscilloscopes. Most often, the charged particles are generated from a hot filament and accelerated through multiple focusing electrodes until they reach a deflecting apparatus whose voltage is changed to control the deflection of the charged particle beam. This deflection is often set up with both x and y controls relative to the display end of the tube where the charged particles eventually impact along a trajectory determined by the deflecting voltage, generating a pixel on the display. The rastering of the electron beam across the display at high frequency allows many of these individual pixels to be drawn in a short time, thus forming an image. Indeed, the deflection of charged particles has played a vital role in everything from mass spectrometry and nuclear fuel refining to display technology and particle science. In fact, the relative ubiquity of charged particle deflection has to some degree hidden its roots in fundamental physics research and its basic value to human knowledge of the quantum world. Historically, J.J. Thompson, a physicist, philosopher and inventor, developed his electron deflection cathode ray tube apparatus in 1897. His experiments consisted primarily of studying charged particles boiled off of a resistive filament through a set of charged accelerating plates through a deflecting electric and magnetic field in an evacuated cathode ray tube as below.

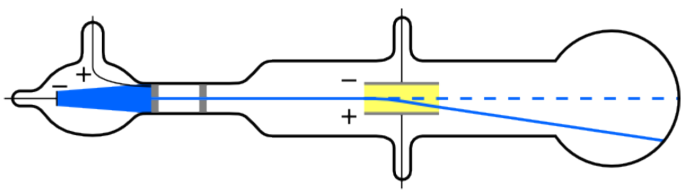


Figure1.)

This apparatus allowed Mr. Thompson to prove definitively that the ever mysterious cathode rays seen in previous experiments were in fact negatively charged particles and to carefully calculate the charge to mass ratio of these particles, electrons, by observing the angle of deflection obtained for a given deflecting voltage.

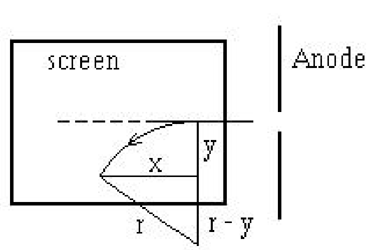
Theory:

General intuition holds that, charged particles will be deflected by electric and magnetic fields. Such intuition takes for granted the details of the discovery, and ultimately the fundamental principles of operation of a large portion of modern technology and basic physics research. Fundamentally, charged particles, such as electrons, undergo deflection due to to the Lorentz force when passing through electric and magnetic fields. This deflection can be restricted to a simpler special case by employing only an electric field. This electric field deflects charged particles, in a direction and with a magnitude according to their charge polarity and intensity respectively; electrons specifically are deflected toward the cathode plate in an electrostatic deflection apparatus such as that designed by J.J. Thompson. A necessary precondition for electrostatic deflection is that the stream of deflected charged particles be of uniform mass, speed and charge, without homogeneity in these characteristics, the particles will tend to scatter and interfere with one another, thus diffusing the cathode ray into a spray of charged particles. In the case of J.J. Thompson's experiment, the charged particles in question are electrons, and their charge to mass ratio was determined as a result of this expected homogeneity of particles, in addition to the clear radius of curvature observed as a direct response of the stream of homogeneous particles to the potential difference between the deflecting plates.

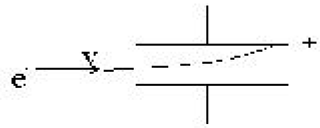
Generally, there exist two known methods for determining the charge to mass ratio of the homogeneous stream of charged particles passing through the Thompson Apparatus. Both require the calculation or observation of the radius of curvature from experimental data. Both methods will be explored herein, and their specific mathematics demonstrated step by step using collected data to prove the validity of both methods for determining the charge to mass ratio, particularly for electrons.

The radius of curvature is calculated the same way for both methods, and can be expressed mathematically and geometrically as shown below:

$$r=\frac{x^2+y^2}{2y} \quad . (1)$$



As discussed, when an electron enters the electric field of the deflector plates, it experiences a Lorentz force, proportional to its charge and the magnitude of the electric field, as shown below:



While the electron remains between the plates, the magnitude of the Lorentz force applied to it is,

$$\sum F = ma = qE = eE \quad (2)$$

and the magnitude of ‘e’ is the elementary charge and ‘E’ the electric field is given by,

$$E = \frac{V}{d} \quad (3)$$

where ‘V’ is potential across the plates and ‘d’ is the distance between them.

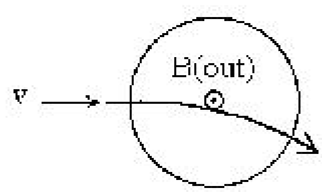
The lorentz force due to the deflecting plates is the primary factor in the original J.J. Thompson apparatus. However, the radius of curvature in this study also included a magnetic field , or B -field, that contributed to the radius of curvature via a pair of Helmholtz coils. These coils provide a magnetic field, perpendicular to the electric field and the motion of the electron beam possessed of a fully tuneable polarity and intensity based on coil current and voltage. Thus, in addition to the deflecting Lorentz force due to the potential across the deflecting plates, the electrons in the beam undergo a gyrating force around the available magnetic field, allowing the path and corresponding radius of curvature to be tuned according the ratio of the strengths of the magnetic and electric fields. mathematically and geometrically, this gyrating force, perpendicular to the electric potential across the deflecting plates and the electron path respectively, appears as a cross product. Fortunately, due to the constant perpendicularity of the B field to the electric field and beam path, this cross product can be reduced to a simple multiplication as below:

$$\sum F = (V * e) \, X \, B \quad (4)$$

which when perpendicular reduces to,

$$\sum F = \frac{m * v^2}{r} \quad (5)$$

which geometrically appears as,



and where the B field is determined by,

$$B = k * I \quad (6)$$

the coulomb constant and current in the coils.

Thus it is apparent from the geometry and direction of the curvature for the electrons, that the electric field and magnetic field can, in correct proportion, cancel out their effects on the electron's path and result in a net path that is virtually indistinguishable from an un-deflected electron. From this relationship between the electric and magnetic fields can the charge to mass ratio of the electron be determined. To calculate the charge to mass ratio of the electron,

$$\frac{v}{B * r} = \frac{e}{m} \quad (7)$$

where 'v' is the velocity of the particle, 'e' is the elementary charge, 'm' is the mass, 'B' is the magnetic field and 'r' is the radius of curvature. However, this relation leaves velocity as an unknown, thus, to find velocity we turn to the electric field, set it equal to the magnetic field, as in the case where their net effects on the path of the electron cancel, and solve thus,

$$\frac{e * V}{d} = v * e * B \quad , (8)$$

with some algebra,

$$v = \frac{V}{B * d} \quad . (9)$$

yielding v in terms of known values.

Now substituting 'v' into equation (7) yields,

$$V=\frac{e}{m}*k^2*I^2*d*r \quad (10)$$

The slope of this variable when plotted against $r*I^2$ yields a linear plot, and with some simplification yields,

$$\frac{e}{m}=\frac{\partial x}{dk^2} \quad (11)$$

where ‘ ∂x ’ represents the slope of the line, the charge to mass ratio is equal to the slope divided by the distance between the deflector plates and the coulomb constant squared.
In this study, the charge to mass ratio turned out to be, (insert here).

As an alternative to the above procedure, the second method for finding the charge to mass ratio of the particles in the J.J. Thompson deflection experiment involves the use of the kinetic energy of the particles and how the potential across the deflector plates effects that kinetic energy. The second method proceeds as follows,

The relationship of kinetic energy to the deflector plate potential is,

$$KE=\frac{mv^2}{2}=eV \quad .(12)$$

Now a constant radius of curvature is found from the B-field strength, the elementary charge, the particle mass, and the velocity of the particle. This radius of curvature is the direct result of the gyration of charged particles in a magnetic field. In this case, specifically for electrons.

$$F=e*v*B=\frac{mv^2}{r} \quad . (13)$$

When solved for electrons specifically, the radius is selected as 0.325 meters. Further, solving for velocity, and re-substituting into the expression for kinetic energy yields,

$$eV=\frac{m}{2}\left(\frac{eBr}{m}\right)^2 \quad .(14)$$

As seen in equation 6, the B field term can be replaced by k*I which, with some algebra readily yields the charge to mass ratio term once more, taking the form:

$$\frac{e}{m}=\frac{2V}{k^2 I^2 r^2} \quad .(15)$$

The charge to mass ratio of the particle, in this case an electron, is found by solving this equation. This operation will yield a series of values for the charge to mass ratio. Taking the average of these values will yield the effective charge to mass ratio for the electron.

Experimental Procedure:

Begin by wiring the correct power supply to the TEL 525 tube and the correct power supply to the TEL 502 Helmholtz coils. A photograph of a correctly assembled experimental setup, along with a wiring diagram are shown in figure 1 and 2 below.

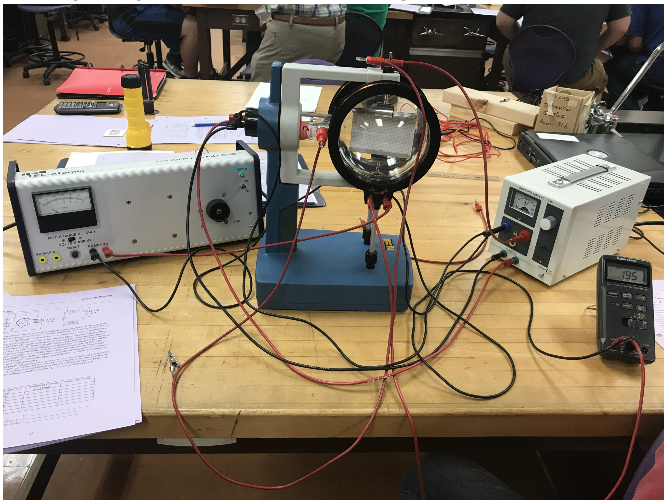


figure 1.)

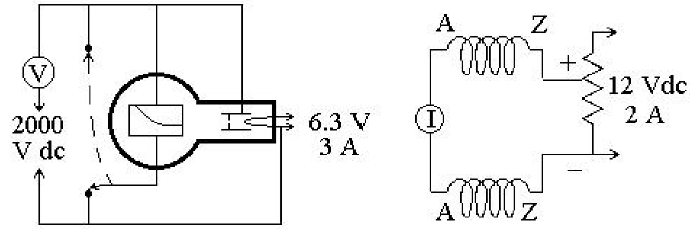


figure 2.)

To implement the first method of determining the charge to mass ratio of the electron, set the voltage of the deflector plates in the TEL 525 tube to 2kv. At this time, a cathode ray of blue or green color should appear on the phosphor display. Manipulate the current through the Helmholtz coils until the electron beam appears at its maximum straightness. In effect, returning to the same horizontal line on the phosphor screen as it appeared to begin at. Once maximum straightness is achieved, without changing any settings, disable the power supply to both the coils, and the deflector plates, and disconnect the leads supplying the potential to the deflector plates. Let the systems discharge briefly, before turning on the power supplies again. A highly bent electron beam should be visible, graph this electron beam and record the coil current, the horizontal and vertical values for where the beam ends, and the voltage. Reattach the leads to the deflector plates and begin again, repeating this process in 500 volt increments all the way up to 4500 volts. Be sure not to exceed the rated voltage of the tube or the rated current of the coils. Excessive current or potential could pose an overload hazard and may result in melting, explosion or electric shock!

In the case of the second method, a constant radius of curvature must be maintained to find the charge to mass ratio of the electron. Thus, begin by setting the deflector plate voltage to 0 volts. Let the anode voltage for the tube begin at 2500 volts and increment up 500 volts per trial until 4500 volts is reached. One must be careful to pick appropriate end points for the electron beam in order to maintain a constant radius of curvature! Thus, at each increment of 500 volts, one must manipulate the current through the Helmholtz coils to adjust the electron beam back into the constant radius of curvature whenever the anode potential changes the beam path. Record the voltages of the anode, the currents of the coils, and the exit points for the electron beam for each trial. The following pictures in figure 3 and 4 show examples of what the tube output should look like for the above procedures.

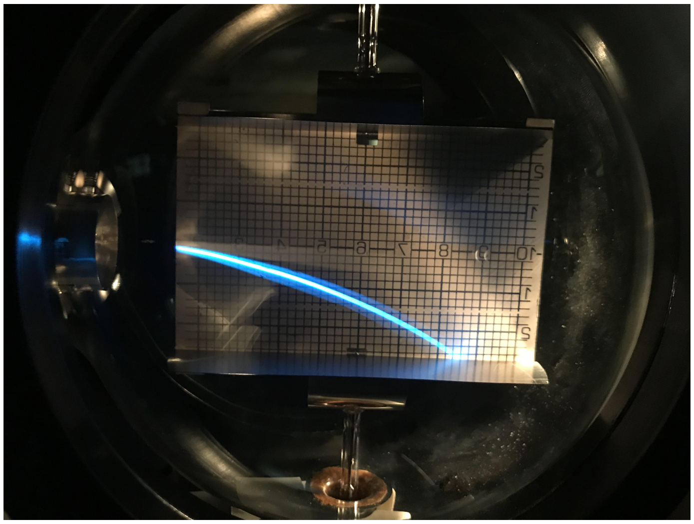


Figure 3.)

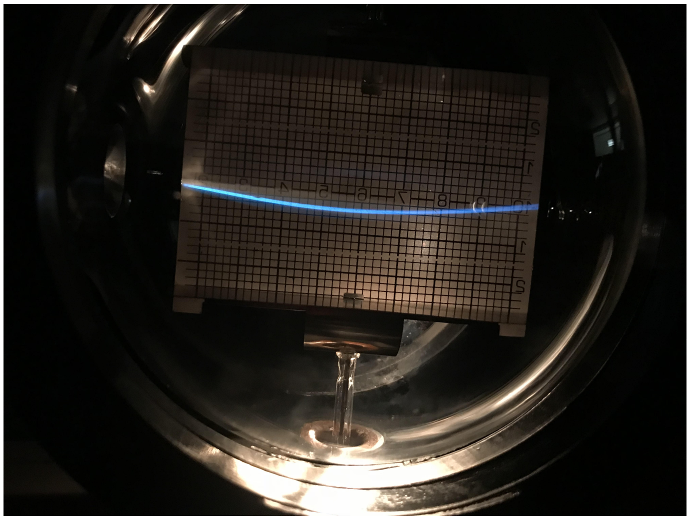


Figure 4.)

Graphs and Charts:

Method 1

Anode Voltage (Volts)	**Helmholtz Current (Amperes)**	**e/m (C/kg)**	**Δx (cm)**	**Δy (cm)**	**r (m)**	**Vp**	**I^2**
2000	0.228	3.04E+11	8.0	2.6	0.136076923	2000	0.051984
2500	0.256	3.01E+11	8.0	2.6	0.136076923	2500	0.065536
3000	0.293	2.76E+11	8.0	2.6	0.136076923	3000	0.085849
3500	0.314	2.80E+11	8.0	2.6	0.136076923	3500	0.098596
4000	0.384	2.14E+11	8.0	2.6	0.136076923	4000	0.147456
4500	0.376	2.51E+11	8.0	2.6	0.136076923	4500	0.141376

The average value of e/m turned out to be 2.71E+11 C/kg.

Method 2

Anode Voltage (Volts)	Helmholtz Current (Amperes)	e/m (C/kg)	Δx (cm)	Δy (cm)	r (m)	I^2
2000	0.225	2.38E+11	8.0	2.6	0.136076923	0.050625
2500	0.256	2.30E+11	8.0	2.6	0.136076923	0.065536
3000	0.292	2.12E+11	8.0	2.6	0.136076923	0.085264
3500	0.323	2.02E+11	8.0	2.6	0.136076923	0.104329
4000	0.371	1.75E+11	8.0	2.6	0.136076923	0.137641
4500	0.375	1.93E+11	8.0	2.6	0.136076923	0.140625
		2.09E+11				

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Calculations:

The equations of r are found by examining the triangles relating r to x and y.

$$r^2 = x^2 + (r - y)^2$$

$$2ry = x^2 + y^2$$

$$r = \frac{(x^2 + y^2)}{2y}$$

Thus is r solved for.

Solving for the contribution of the electric force is shown:

$$F = ma = eE$$

$$E = \frac{V_p}{d}$$

$$eE = \frac{(eV_p)}{d}$$

Solving now for the contribution of the magnetic force:

$$B = kI$$

$$\left(\frac{e}{m}\right) = \frac{v}{Br}$$

The charge to mass ratio derived from the magnetic force will be used to solve for V_p in method one, substituting a value for v and B. One can solve for v by setting the two forces equal.

performing a substitution…

$$\frac{e}{m} = \frac{V_p}{B^2} dr$$

$$V_p = \frac{e}{m} B^2 dr$$

$$V_p = \frac{e}{m} k^2 I^2 dr$$

This concludes method one.

Method two is shown below:

$$F = veB = \frac{mv^2}{r}$$

$$eB = \frac{mv}{r}$$

$$v = \frac{eBr}{m}$$

Substitute the above into the kinetic energy equation.

$$KE = \frac{mv^2}{2} = eV_a$$

$$\frac{m}{2}\frac{eBr}{m^2} = eV_a$$

Now solve for e/m...

$$eV_a = \frac{me^2 B^2 r^2}{2m^2}$$

$$v_a = \frac{eB^2 r^2}{2m}$$

$$e/m = \frac{2V_a}{B^2 r^2}$$

Last, Substitute for B...

$$e/m = \frac{2V_a}{k^2 I^2 r^2}$$

Analysis and Conclusions:

The values of e/m found by using the slope of the graphs found from methods one and two were 1.82x10^11 C/kg and 1.5x10^11 C/kg, respectively. These values vary above and below the expected value of 1.75x10^11 C/kg, by 4.00% and 14.29% respectively; Variations could be attributed to equipment age, measurement error, and natural variances in local electromagnetic fields, in vicinity of the experiment. Further, if the tolerances in the manufacturing of the TEL525 tube are sub par, the plate spacing could be off resulting in error. Additionally, electron beams are not perfectly uniform and spread as they travel resulting in some natural variance. All calculations were made with fundamental laws of electromagnetism and geometric relationships, thus systemic errors and natural variance are the primary sources of error in the measured values.

The data was gathered via photography and readouts from the built in monitors on the instrumentation in the lab. The electron tube was prone to "sputtering" at higher voltages, which could also have contributed to the error experienced. Furthermore, the sputtering contributed to unreliability in the photographic data as the picture captures only a single moment in time, effectively grazing over the time varying signal produced by the sputtering phenomenon. Ideally, multiple photos would be taken of the sputtering beam allowing a time averaged series to be built, but the available photographic systems did not allow for efficient and reliable time series photography. As is apparent from the relatively small error, the unreliability of the photographic data and sputtering phenomenon did not ultimately damage the precision or accuracy of the data beyond usable bounds. Given the age of the equipment and the relative lack of sophistication of the photographic systems available, the percent error is quite small, and the precision and accuracy of the measured charge to mass ratio is excellent. In fact, averaging the two values found yields a value within 5% of the accepted value for the charge to mass ratio of an electron, very close indeed.

The measurement of the charge to mass ratio is informative as it is a natural foundation to discovering the deeper nature of the charged particles in cathode rays. As J.J. Thompson in the late 1800s did, the discovery of cathode rays and their particulate nature led to further exploration of the negatively charged particle constituents, eventually to be known as electrons. With the official discovery of the existence and properties of the electron, the age of electronics, and of quantum mechanics could begin, birthing the cornerstone of a wide array of modern technologies on which society, and humanity now depend.

Bibliography: