Modern Physics: Applying Statistical Analysis to Radioactive Decay

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Abstract:

The investigator applied statistical analysis techniques to radioactive decay data gathered via a Geiger-Muller tube. The investigator mapped out the plateau and operational voltage of the Geiger Muller tube by iterating and recording counts in 20volt increments until the upper knee was reached. The lower knee, operational voltage and upper knee were, 780volts, 900 volts, and 1020 volts respectively. Later, spreadsheet programs and other data analysis tools were employed on the gathered count data to develop familiarity with statistical procedures such as standard deviation, error, variance, and to glean understanding of the effects of such figures in the laboratory environment. The results were found to correlate strongly with historical expectation values.

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Introduction:

The investigator's purpose in the experiment was to extract data from a highly stochastic process using statistical analysis techniques and good laboratory practices. Additionally the researcher pursued understanding of their laboratory equipment, its limitations, and the consequences thereof to the collection and analysis of experimental data via repeated collection runs with the Geiger-Muller tube(apparatus). Collection runs were performed in 100 volt increments until the lower knee of the detection curve for the tube was reached. Next, 20 volt increments where used to increase the effective resolving power of the apparatus over the plateau range, where the delta in counts per voltage step is much smaller. Voltage was continuously raised until counts began to climb exponentially again, thus marking the upper knee of the detection curve. Data points were continuously recorded and stored for later processing. In service of the task, precision error measurement, deviation, sample mean and variation calculations were performed on the data collected via the Geiger Muller tube in a spreadsheet calculation system.

The work done by the investigator would be impossible without the use of the Geiger-Muller tube apparatus; thus, a discussion of its history and function is in order. Sometime in 1908, Geiger and Rutherford et al. Published a paper outlining a method of detecting radiation via electron cascades, known as Townsend avalanches, between highly charged conductors in a gas filled chamber. This discovery led eventually to an additional publication in 1928 by Geiger and Muller implementing the concept in a more useful form factor that would allow more wide spread application of the "electronic radiation counting device" in research and industry. At its foundation, the Geiger-Muller tube is a set of electrodes held generally at several hundred volts in a rarefied gas atmosphere. When radiation penetrates the containment and ionizes the rarefied gas, it generates fast moving liberated electrons that in turn ionize more gas molecules, freeing even more electrons. Ultimately these electrons are drawn to the high positive potential on the central anode and form an electron cascade of sufficient magnitude to disturb the steep voltage potential between the electrodes and in turn the voltage across a measurement resistor in the detection device. The disturbances are then counted and translated to a audible beeping or clicking sound by the detector informing the operator of the approximate radioactive state of their surroundings. In relation to the investigator's experiment, the operation and development of the Geiger-Muller tube explains the nature of the "count" variable and aids in understanding the nonlinear nature of the detection curve. Additionally, the role that the electric potential(voltage) plays in the operation of the tube illuminates some of the limitations of the apparatus and is enlightening as to the relationship of the voltage and detection sensitivity, as well as to the interesting, but potentially dangerous effects of operating in environments where the Geiger-Muller apparatus runs up against its functional limits.

Theory:

Preempting the exploration of radiation and statistical analysis of a stochastic process, is understanding the theory behind statistics and radiation themselves. A common feature of stochastic processes is the need for repeated measurements and relatively large sample sizes recorded under reasonably controlled conditions and in close cropped time intervals in order to produce curves that most closely fit collected data. However, even under perfectly controlled conditions it is impossible to precisely predict the outcome of a stochastic event, especially in the case of individual decaying particles. In reality, large collections of radioactive particles are the dominant state in which they are encountered, thus, statistical estimation becomes an effective method for predicting average radioactivity. The arithmetic mean is the preferred tool for measuring average activity in radioactive samples it is denoted (n’). It is calculated as:

$$n' = \frac{\sum n}{N}$$

where the sum of n is all counts over a time interval, and ‘N’ is the total number of observations. Theoretically, the sample mean is the true mean only when an infinite number of measurements are made. In reality, measurements can only be made for a finite time and thus, error accounting and correction becomes necessary.

Generally, error takes two forms, determinate or indeterminate; Where determinate errors encompass faulty experimental technique, miscalibrated equipment, and systematic failures. Determinant errors can be mitigated by careful planning, meticulous organization and cautious experimentation on the part of the investigator. Indeterminate errors however, can not be controlled by laboratory procedure or investigator caution, as they are often of a random nature and defy effective prediction. Radioactive decays would be just one source of such indeterminate errors, as their stochastic nature precludes prediction and complicates accurate study of individual radioactive particles. As many learned in their early education, for any measurement to be accurate, it must be unbiased and precise. That is, the measurements must be consistent from sample to sample, as measured by the “standard deviation” in order to account for indeterminate errors as well as being either voluminous or of sufficient quality to counteract effects of machine miscalibrations or failure of analytical equipment or personnel. Concisely, accuracy is proximity of a measured value to a known or preferred value; while precision is the grouping of any set of values relative to the surrounding measurements.

Often, in discussions of precision and accuracy, analysis of the data turns to probability as the fundamental feature for determining distribution of data sets. As defined, probability is the likelihood of an occurrence denoted as a fraction between zero, and one. If an event has a probability closer to zero, then it is a less likely occurrence; as an event trends closer to one, then it is a more likely occurrence. Such probability values can be plotted and evaluated as a graph using the Poisson function, below:

$$P_n = \frac{u^n e^{-u}}{n!} \quad .$$

The Poisson function generates a probability distribution curve that depicts the likelihood of a random event. Its flexibility is such that it is applied across sciences, from describing chemical compound interaction rates, to radiosource decay rates and quantum interaction cross sections. The Poisson distribution is a potent statistical tool, but it is computationally complex. A similar effect to the Poisson function can be achieved with the “Gaussian” function shown below:

$$G_n = \sqrt{\left(\frac{1}{2\pi u}\right)} e^{-\left(\left((n-u)^2\right)/2u\right)}$$

This function can be used to more quickly obtain a probability description of the counts produced by a radioactive source, as compared to the description of the Poisson function.

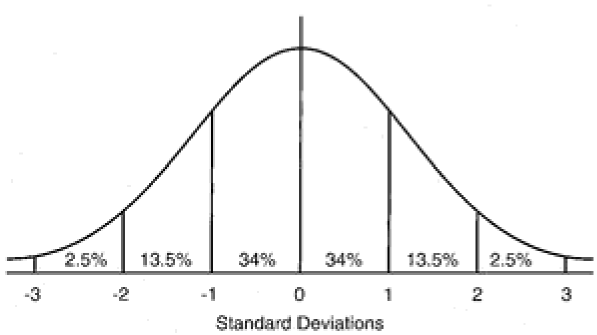
In order to develop a complete grasp of data collection and analysis, it is necessary to discuss error and its calculation to promote fidelity in the reporting of laboratory data and inform further analysis of the specific radiometric data gathered via the geiger-muller tube. Relative error can be calculated by finding the number of standard deviations in the error, like so,

$$n - u = \tau\sigma$$

Then, the probability of that error falling within expectation values, or the “confidence interval”, can be calculated with the relation,

$$G_\tau d\tau = \frac{1}{\sigma\sqrt{(2\pi)}} e^{((-\tau^2)/2)} dt \quad .$$

This is a function in terms of sigma, or standard deviations. Any measurement of confidence will be given in terms of sigma, or the confidence interval, denoting a quantification of the level of confidence in a particular measurement, based on its relation to other measurements in a data set, and its accuracy when compared to expected values. A similar system can be observed in use in high energy physics experiments, as well as observations of nuclear decay. For instance, CERN, the European nuclear research institution regularly updates the public as to discoveries of new particles, however, such announcements are made only when the sigma of a measurement has exceeded a certain confidence level. CERN performs hundreds of thousands or even millions of particle collisions per day in a known set of energy levels. Whenever an as yet unpredicted or unseen energy signature appears in the particle collision cloud remnant data usually in the form of a bump in the debris cloud energy curve, more experiments in that energy domain are performed to draw out repeated occurrences and build up a profile of a phenomenon at a given energy range, in terms of its probability of occurrence and the significance of the measurement. As more data builds up, and the occurrence repeats or does not repeat for a given collision energy, a confidence interval and set of error bounds are produced to describe the likelihood that the discovery is a real particle and worth reporting. Once the occurrence has been mapped multiple times with in the expected error bounds and is shown to be an event with known regularity at a given collision energy, a confidence interval can be established along with a gaussian or poisson distribution of its production probability at that energy level. If the event continues to be mapped within expected error bounds for that collision energy, the confidence interval grows, and certainty of the discovery increases, that is the “sigma” of the event increases, which reduces the ratio of the uncertainty and moves the probability of the event being real closer to one. An example of a Gaussian distribution with standard deviations labeled is below.



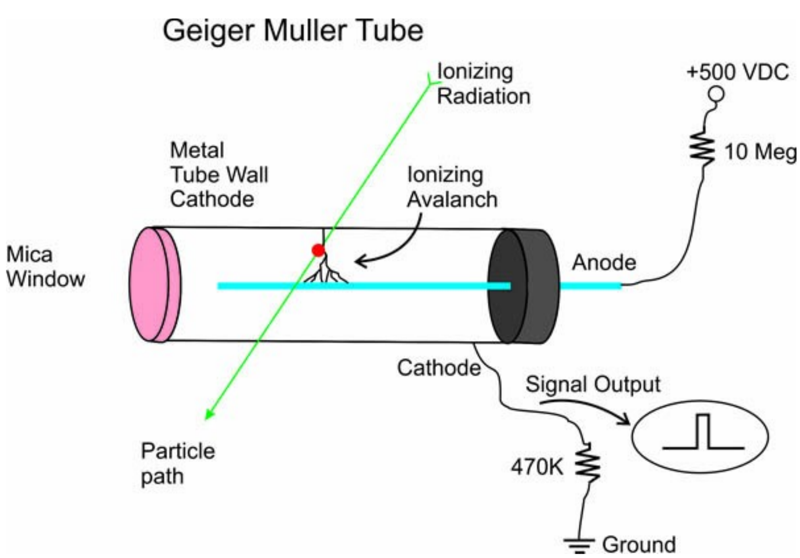
As seen in the graph, any measurement made for its data set will be within one standard deviation of the mean value 34% of the time, that is, you can be 34% confident of a measurement landing within one standard deviation of the mean. Additionally, one can see that 95% of the area under the curve is contained within 2 standard deviations of the mean, showing that the measurements will be within 2 standard deviations 95% of the time. Beyond just telling you the confidence and probability of a particular dataset, the gaussian and poisson distributions can inform investigators of the “probable” error, which is a description of the 50% confidence interval in a dataset, defined mathmatically;

$$P = 0.6745\sigma \quad .$$

One might expect data to conform to a normal curve, such as the gaussian or poisson distribution, but as all real experiments are constrained to approximating the mean in finite time, and the standard deviation relative to that mean, exactitude cannot be assumed. In reality, the approximated mean for finite time, n’ and the sample standard deviation “s” are used. S can be given mathematically as,

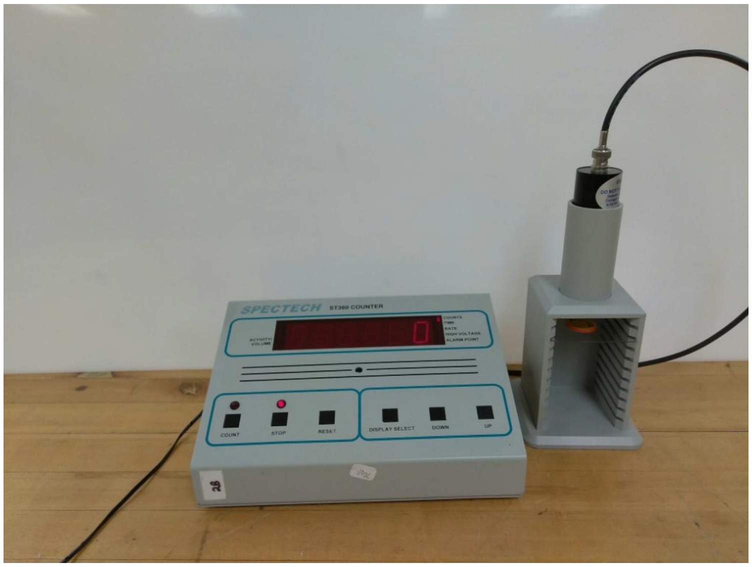
$$s = \sqrt{\left(\frac{1}{N-1} / \sum (n - n')^2\right)} \quad .$$

With the conclusion of the discussion of statistical theory and analysis, begins the exploration of the operational theory specific to radiation detection and counting, and how the data sets, to which statistical analysis is applied, are collected. Radiation can be characterized as energized particles such as electrons, alpha particles, and photons, that, when they interact with other matter tend to deposit their energy into the particles of that matter, resulting in myriad effects, from ionization, to nuclear reactions, decay events, electric currents and heat. In the case of the Geiger-Muller apparatus, energized particles, pass through a thin “window” into a charged tube containing a thin charged wire and a certain amount of gas, usually helium or argon, but sometimes radon or xenon, at low pressure. When the energized radiation, whether electrons, alpha particles or photons, interacts with the gas molecules in the tube, the gas atoms sometimes become ionized and are drawn along the established voltage gradient in the tube, toward the outer wall. As they travel, the free electrons from other atoms and increase the number of liberated electrons in the tube. These liberated electrons travel at high speed along the established voltage gradient toward the center filament, sometimes freeing even more electrons as they travel. This electron cascade is referred to as a townsend avalanche and eventually results in a substantial spike in the current on the center filament. This process allows individual rays of radiation at sufficient energy to trigger short pulses of current which can be counted by the counting machine in order to establish the intensity of the radiation measured. Thus, “counts” are found in relation to the radiation intensity and the voltage of the tube. The process of ionization of the gas in the tube results in the accumulation of positive ions at the cathode(tube wall), and electrons at the anode(center filament) which, if left unchecked would result in continuous discharge and the ruination of the Geiger-Muller apparatus. To combat this problem, Geiger-Muller devices have various modes of “quenching”, either by intermittently altering the voltage gradient in the tube, or through the addition of halogens or poly-atomic gases to the rarefied atmosphere to allow recombination of electrons with positive ions and allow recovery of steady state of charge even after large numbers of ionizing interactions with radiation sources. A depiction of the Geiger Muller tube is shown below.



Experimental Procedure:

An experimental setup as shown was assembled from available equipment including, a Geiger Muller tube and counter, tube holder, test rack and radioactive samples. The counter was plugged into 120 volt AC power and connected via coaxial cable to the Geiger-Muller tube. The Geiger-Muller tube was carefully, especially with respect to the mica detection window, placed into the aperture of the sample rack, and hooked up to the counter on the appropriate port. The counter was then calibrated for 0 volts of control voltage, and 30 second intervals for the first trial.



A gamma ray producing sample was placed on the top shelf of the sample rack and the Geiger-Mueller tube placed in the aperture above the shelf. The investigators adjusted the control voltage by one hundred volts per trial, and recorded the number of counts read out every 30 seconds. Eventually, the plateau voltage of the Geiger-Muller tube was achieved and the investigators reduced the voltage down to the initial increment.

The investigators then began to map the plateau response of the Geiger-Muller tube in continuous 20 volt control increments. No disturbances of the Tube, counter or source were made throughout the experiment, until a catastrophic failure of a gas line outside the laboratory necessitated the evacuation of all personnel and cut the experiment short. The gathered data, along with an example of statistical analysis on an equivalent but separate data set are included below.

Graphs and Charts:

General Statistical Analysis Data: chart(1)

Trial No.	n	|n -`n|	(n -`n)²	(n - `n)
1	2,275.00	47.22	2,229.73	47.22
2	2,227.00	0.78	0.61	-0.78
3	2,374.00	146.22	21,380.29	146.22
4	2,463.00	235.22	55,328.45	235.22
5	2,333.00	105.22	11,071.25	105.22
6	2,276.00	48.22	2,325.17	48.22
7	2,453.00	225.22	50,724.05	225.22
8	2,396.00	168.22	28,297.97	168.22
9	2,298.00	70.22	4,930.85	70.22
10	2,365.00	137.22	18,829.33	137.22
11	2,337.00	109.22	11,929.01	109.22
12	2,337.00	109.22	11,929.01	109.22
13	2,293.00	65.22	4,253.65	65.22
14	2,301.00	73.22	5,361.17	73.22
15	2,087.00	140.78	19,819.01	-140.78
16	2,122.00	105.78	11,189.41	-105.78
17	2,159.00	68.78	4,730.69	-68.78
18	2,130.00	97.78	9,560.93	-97.78
19	2,180.00	47.78	2,282.93	-47.78
20	2,041.00	186.78	34,886.77	-186.78
21	2,121.00	106.78	11,401.97	-106.78
22	2,082.00	145.78	21,251.81	-145.78
23	2,218.00	9.78	95.65	-9.78
24	2,234.00	6.22	38.69	6.22
25	2,155.00	72.78	5,296.93	-72.78
26	2,170.00	57.78	3,338.53	-57.78
27	2,230.00	2.22	4.93	2.22
28	2,515.00	287.22	82,495.33	287.22
29	2,531.00	303.22	91,942.37	303.22
30	2,448.00	220.22	48,496.85	220.22
31	2,346.00	118.22	13,975.97	118.22
32	2,364.00	136.22	18,555.89	136.22
33	2,282.00	54.22	2,939.81	54.22
34	2,110.00	117.78	13,872.13	-117.78
35	2,100.00	127.78	16,327.73	-127.78
36	2,197.00	30.78	947.41	-30.78

37	2,263.00	35.22	1,240.45	35.22
38	2,060.00	167.78	28,150.13	-167.78
39	2,095.00	132.78	17,630.53	-132.78
40	2,171.00	56.78	3,223.97	-56.78
41	2,079.00	148.78	22,135.49	-148.78
42	2,126.00	101.78	10,359.17	-101.78
43	2,060.00	167.78	28,150.13	-167.78
44	2,151.00	76.78	5,895.17	-76.78
45	2,121.00	106.78	11,401.97	-106.78
46	2,192.00	35.78	1,280.21	-35.78
47	2,081.00	146.78	21,544.37	-146.78
48	2,118.00	109.78	12,051.65	-109.78
49	2,114.00	113.78	12,945.89	-113.78
50	2,208.00	19.78	391.25	-19.78
Sum	**111,389.00**	**5,405.68**	**818,442.58**	**0.00**
Average	**2,227.78**	**108.11**	**16,368.85**	**0.00**
Standard Deviation	**129.24**	**69.11**	**19,629.63**	**129.24**
Sample Variance	**16,702.91**	**4,775.82**	**385,322,278.69**	**16,702.91**

Gathered Sample Radiation Data: (chart 2)

Voltage	Counts
100	0
200	0
300	0
400	0
500	0
600	0
680	294
700	420
720	363
740	378
760	367
780	443
800	422
820	426
840	460
860	446
880	464
900	456
920	474
940	506
960	525

	980	559
	1000	584
	1020	607
	1040	1089
	1060	3151
Sum	**19500**	**12434**
Average	**750**	**478.23**
	259.923065	**600.281887**
Std. Devi.	**540556**	**628958**
		360338.344
Variance	**67560**	**615385**

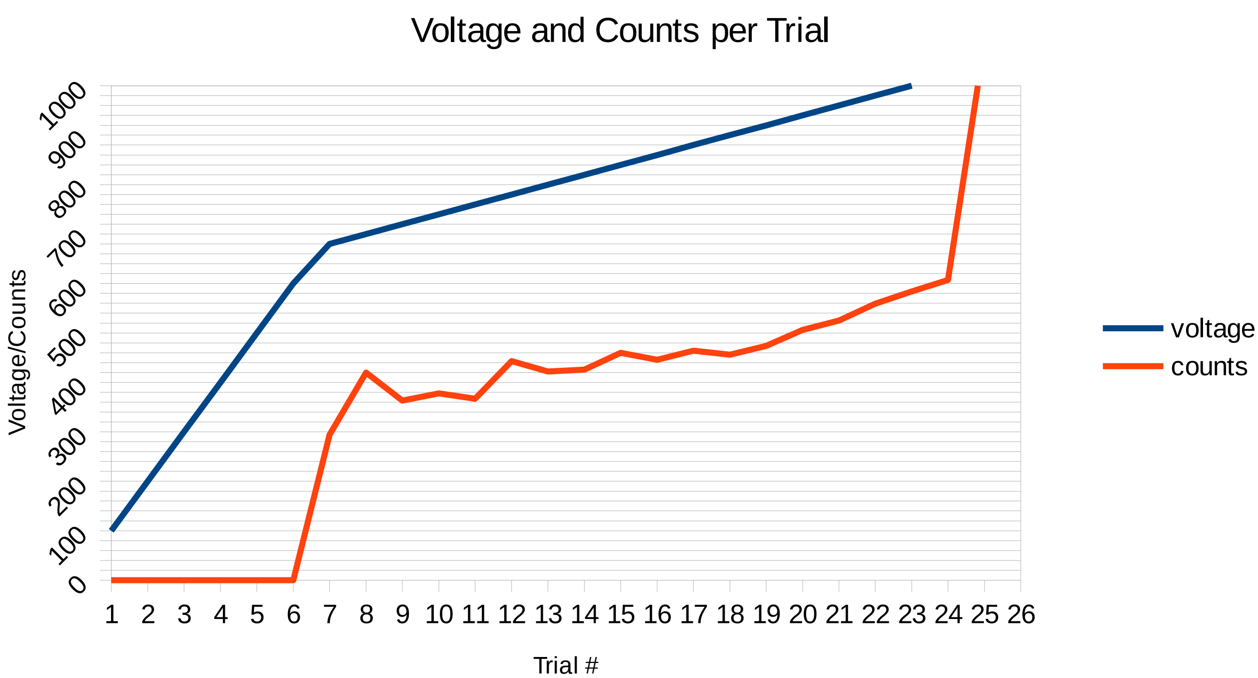


Figure 1.)

A clear plateau is shown in the above graph of the radiation data. It begins at roughly 740 volts and continues past the operational voltage of 900 volts and on up to the beginning of the upper knee at 1000 volts. The mean count value can be seen in the second chart above, on which the graph is based, to be 478.23 counts per trial, with a substantial deviation of 600.28~ counts.

Calculations:

$$n' = \frac{\sum n}{N}$$

$$P_n = \frac{u^n e^{-u}}{n!}$$

$$G_n = \sqrt{\left(\frac{1}{2\pi u}\right)} e^{-\left(\left((n-u)^2\right)/2u\right)}$$

$$n - u = \tau\sigma$$

$$G_\tau d\tau = \frac{1}{\sigma\sqrt{(2\pi)}} e^{\left((-\tau^2)/2\right)} dt$$

$$P = 0.6745\sigma$$

$$s = \sqrt{\left(\frac{1}{N-1} / \sum (n-n')^2\right)}$$

Analysis and Conclusions:

The results of the radiation counting experiment proved to fit expectations nicely. The lower knee, plateau, operating voltage and upper knee of the Geiger-Muller tube were within expectation values for such a device in its condition and for its age. The sample of radioactive cobalt, a gamma source, yielded counts of 464-474 counts in center of the operating voltage range for the Geiger-Muller tube. In the case of the radioactive sample data, the full set of 50 trials is unavailable, thus causing some irregularities in the plateau due to a sparse sample set. The experiment was cut short roughly half way through data collection, due to a major gas leak. Fortunately, the equipment was not disturbed during the 26 intervals in which data was collected, thus eliminating the need to manipulate any data to better fit expectations or perform heavy error correction. The collected data covered the full operating range of the Geiger-Muller tube, and so was sufficient for analysis and plotting. Additionally, the calculated mean, 478.23, of the counts is reasonable for the range of expected values and closely matches the counts, 464-474, registered in the operational voltage range of the Geiger-Muller tube, well within one deviation of the mean. Furthermore, in the data that was collected, very little error is present; any error that can be found would be due to the age of the equipment, wear and tear on the electronics of the counter or tube, or perhaps due to fluctuations in power supply. Regardless of the source, all error appeared to be sub threshold and did not meet the criterion of significance. On the other hand, variances in the data were extreme, as testified by the value in chart 2, 360338.34~; This large value can be explained by the substantial unpredictability of radioactive decay and a small number of trials. Decay events will be on average, of mean intensity and timing, however, there will exist many events in any mass of decaying atoms that occur multiple deviations from the mean which contributes substantially to the variance in the data. While this variance is not apparent in the graph due to the small number of samples, given a larger time to sample and a larger volume of trials, the variance would make itself more apparent in the data and would likely normalize with increasing number of samples. On the statistical analysis data, everything appears to be in its proper place, error values are contained because the data was gathered in a controlled environment with reliable equipment and no major mishaps. Additionally, the variance in the data set in chart 2 is seen to be much lower than for the measured radioactivity data, due to the longer exposure times and the larger number of trials; as expected, the variance normalizes with increasing numbers of trials. Furthermore, the average counts in the statistical analysis data appears to fit expectation values for 1 minute exposure times and the radioactive source used(also cobalt). The standard deviation for the data set is small due to the larger number of samples and longer exposure times. With more time to view the sample, the tube and counter are able to reach a more stable value of radiation counts, and with more trials, the deviations in the data shrink relative to the quantity of samples collected, thus reducing the standard deviation of the data. Whats more is, the statistical data appear to have controlled sample variance due to the larger number of trials relative to the observed variance in each sample.

In conclusion radioactive sources do in fact radiate in highly variant and unpredictable ways. Decay events remain unpredictable at an individual level, in much the same way that classical mechanics breaks down when attempting to calculate the motions and positions of more than 3 bodies simultaneously. The sheer number of decay events happening simultaneously in even a tiny amount of appreciably radioactive material makes preempting decay events impossible; However, the intelligent application of statistics to carefully collected data in sufficient volumes can allow for some degree of understanding and analysis of an otherwise difficult system. By reducing the problem to a set of averages, and utilizing the tools of statistics, such as deviation, mean, median, mode, error, and variance one can extract useful information about whole radioactive sources and determine pragmatic details about those sources, their safety and their usefulness. Additionally, one ought to know the limits of ones tools, such as the limits of the Geiger-Muller tube; The Geiger-Muller apparatus is blind for a brief period after it encounters an ionizing particle due to the imbalance of charges in the tube, thus a quenching system is required to restore charge balance. However, any quenching system will have an upper limit on the rate at which it can restore the charge balance in the tube, thereby placing a limit on the maximum radiation dosage a specific tube can reliably count. In fact, if that maximum of reliability is exceeded, the tube may begin to backtrack producing erroneous readings of lower radiation levels than those that exist in the measured sample. Hence, why many modern Geiger-Muller based radiometric devices include feedback and control circuitry that can warn of a over dosage failure or measured radiation counts in excess of the reliability limits.

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Image of Geiger-Muller Tube:
http://www.imagesco.com/articles/geiger/build_your_own_geiger_counter_-_gm_tube.html

Credit Yael Mckorkle for the Image of experimental Setup.